Reusing Learning Objects via Theory Morphisms

Michael Kohlhase, Marcel Schütz

FAU Erlangen-Nürnberg

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Learning Objects

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What is a Learning Object?

Theorem. Propositional logic $\langle \models_{\mathsf{PL}^0}, \mathcal{L}_{\mathsf{PL}^0}, \mathcal{K}_{\mathsf{PL}^0} \rangle$ naturally forms a logical system $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$.

Definition. Let A be a formula of propositional logic. A is satisfiable iff there exists a model $\mathcal{I}$ such that $\mathcal{I}(A) = \mathsf{T}$.

Exercise. Is the formula $p \wedge (q \Rightarrow \neg p)$ satisfiable?

☐ Yes

☐ No

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Generalization and Exemplification

Definition. A logical system is a triple $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$, where

- $\mathcal{L}$ is a formal language, whose elements are called formulas,
- $\mathcal{M}$ is a set, whose elements are called models,
- $\models \subseteq \mathcal{M} \times \mathcal{L}$.

Definition. Propositional logic is the triple $\langle \models_{\mathsf{PL}^0}, \mathcal{L}_{\mathsf{PL}^0}, \mathcal{K}_{\mathsf{PL}^0} \rangle$, where

- $\mathcal{L}_{\mathsf{PL}^0}$ is the set of all propositional formulas,
- $\mathcal{K}_{\mathsf{PL}^0}$ is the set of all Boolean interpretation functions,
- $\mathcal{I} \models_{\mathsf{PL}^0} A$ iff $\mathcal{I}(A) = \mathsf{T}$.

Theorem. Propositional logic $\langle \models_{\mathsf{PL}^0}, \mathcal{L}_{\mathsf{PL}^0}, \mathcal{K}_{\mathsf{PL}^0} \rangle$ naturally forms a logical system $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$.

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Recontextualizing Statements

Logical Systems:

Definition. Let F be a formula of a logical system. A model $\mathcal{M}$ satisfies F iff $\mathcal{M} \models F$.



Propositional Logic:

Definition. Let A be a formula of propositional logic. A model $\mathcal{I}$ satisfies A iff $\mathcal{I} \models_{\mathsf{PL}^0} A$.

Definition. Let A be a formula of propositional logic. A model $\mathcal{I}$ satisfies A iff $\mathcal{I}(A) = \mathsf{T}$.

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Recontextualizing Exercises

Logical Systems:

Exercise. Is the formula $F := \texttt{definiens}(F)$ satisfiable?
☐ Yes
☐ No
Feedback: Actually, there is a model $\mathcal{M}$ that satisfies F:
$\texttt{definiens}(\mathcal{M})$. Then $\texttt{conclusion}(\Phi)$:
Indeed, $\texttt{proof}(\Phi)$.

Propositional Logic:

Exercise. Is the formula $A := p \wedge (q \Rightarrow \neg p)$ satisfiable?
☐ Yes
☐ No
Feedback: Actually, there is a model $\mathcal{I}$ that satisfies A:
It maps p to T and q to F. Then $\mathcal{I} \models_{\mathsf{PL}^0} A$:
Indeed, this can directly be seen by evaluating the truth table for A.

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Theory Morphisms

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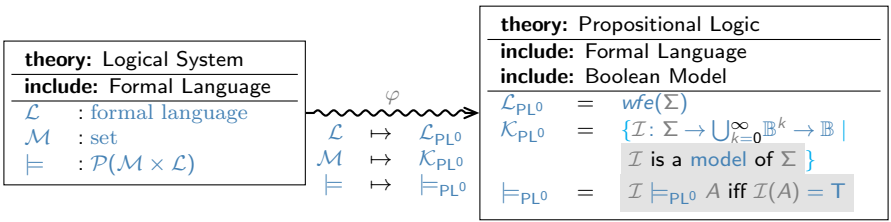
What is a Theory Morphism?

Definition. A theory morphism $\varphi : S \rightsquigarrow T$ between two theories S and T is a mapping of the symbols in S to expressions in T such that

$$s : \tau \quad \Longrightarrow \quad \varphi(s) : \varphi(\tau)$$

for all symbols s and types τ in S.



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Representing Theory Morphisms in sTEX

```
\begin{mathstructure}{logsys}[logical system]
  ...
  % The satisfaction relation:
  \symdef{satrel}[
    args=2,
    type={\powerset{\cart{\modcls,\flang}}}
  ]{#1\vDash#2}
  ...
\end{mathstructure}
```

```
\begin{mathstructure}{proplog}[propositional logic]
  ...
  % The satisfaction relation:
  \symdef{psat}[
    args=2,
    def={\textrm{$\psat{\Ivar!}{\Avar}$ iff $\eq{\Ivar{\Avar},\semtrue}$}}
  ]{#1\vDash_{\textrm{PL}^0}#2}
  ...

  % The theory morphism from logical systems to propositional logic:
  \begin{realization}{logsys}
    ...
    \assign{satrel}{\psat!}
    ...
  \end{realization}
\end{mathstructure}
```


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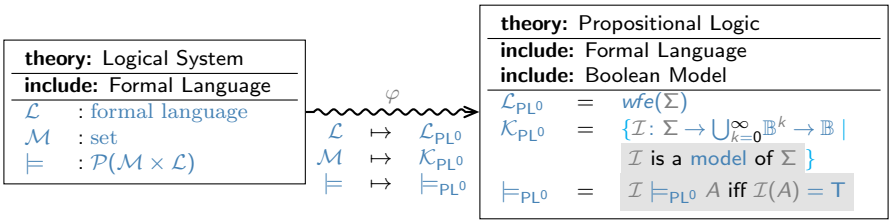
Presenting Theory Morphisms



Theorem. ⟨target concept⟩ ⟨target structure⟩ naturally forms a ⟨source concept⟩ ⟨source structure⟩.

Theorem. Propositional logic $\langle \models_{\mathsf{PL}^0}, \mathcal{L}_{\mathsf{PL}^0}, \mathcal{K}_{\mathsf{PL}^0} \rangle$ naturally forms a logical system $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$.

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Recontextualizing Learning Objects Along Theory Morphisms

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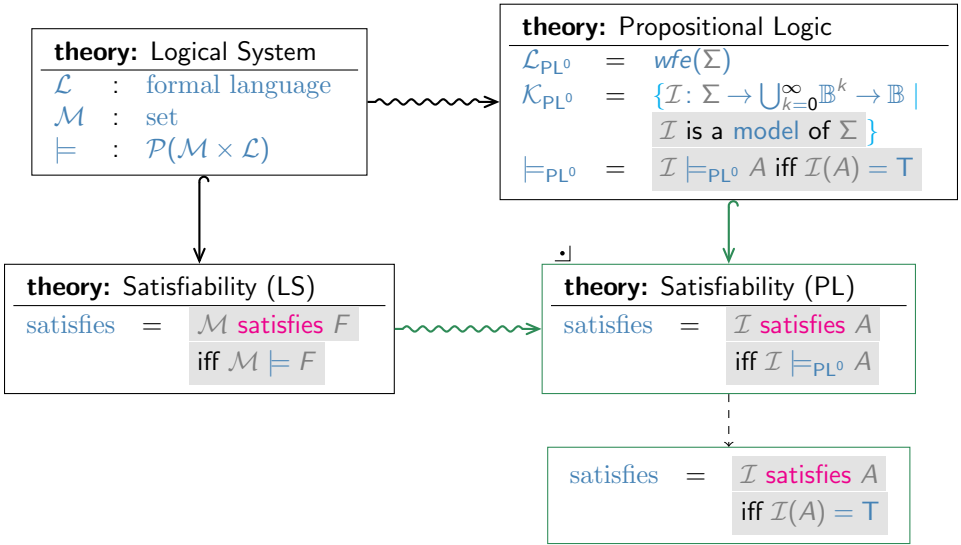
Recontextualizing Statements (Revisited)



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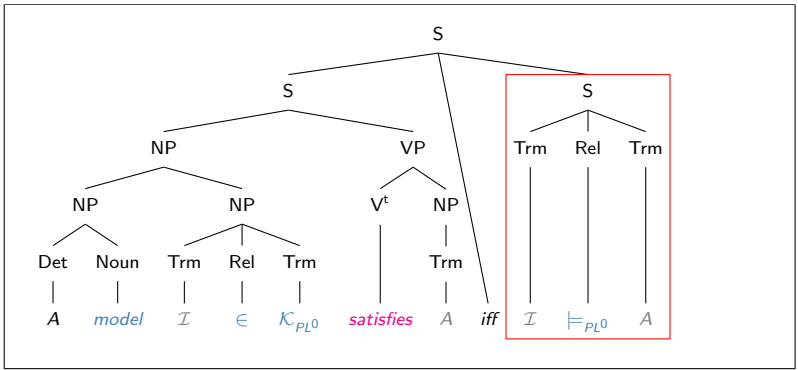
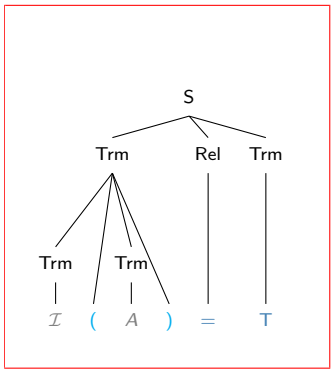
Definition Expansion

"A model $\mathcal{I} \in \mathcal{K}_{\mathsf{PL}^0}$ satisfies A iff $\mathcal{I} \models_{\mathsf{PL}^0} A$."





"A model $\mathcal{I} \in \mathcal{K}_{\mathsf{PL}^0}$ satisfies A iff $\mathcal{I}(A) = \mathsf{T}$."

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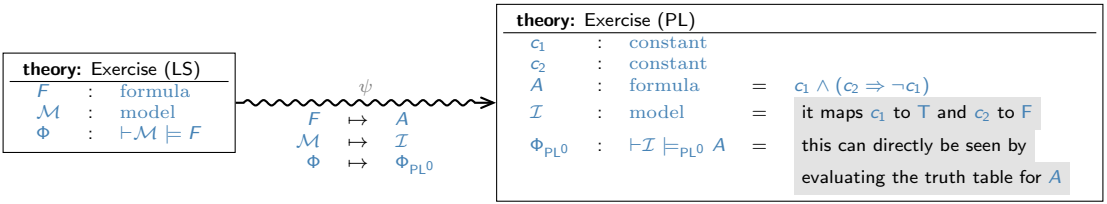
Recontextualizing Exercises (Revisited)



Exercise. Is the formula $F := \texttt{definiens}(F)$ satisfiable?

☐ Yes

☐ No

Feedback: Actually, there is a model $\mathcal{M}$ that satisfies F:
$\texttt{definiens}(\mathcal{M})$. Then $\texttt{conclusion}(\Phi)$:
Indeed, $\texttt{proof}(\Phi)$.

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Conclusion
and
Future Work

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Reusing Learning Objects via Theory Morphisms

- We can **automatically** . . .
  - . . . present theory morphisms in natuaral language.
  - . . . recontextualize statements along theory morphisms.
  - . . . recontextualize exercises with solutions and feedback along theory morphisms.

  ⟶ https://gitos.rrze.fau.de/voll-ki/fau/system/relocalization/

- We have a growing corpus of theory morphisms and templates that are suited for automatic recontextualization.
  ⟶ E.g. https://gl.mathhub.info/courses/FAU/AI/problems/-/tree/main/source/csp/prob

- **Future Work:**
  - Properly handling the intricacies of natural language.
  - Integrating the recontextualization processes in our adaptive learning assistant ALeA.
    ⟶ https://courses.voll-ki.fau.de/

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