Managing Naproche Formalizations with sTeX and FLAMS

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Overview

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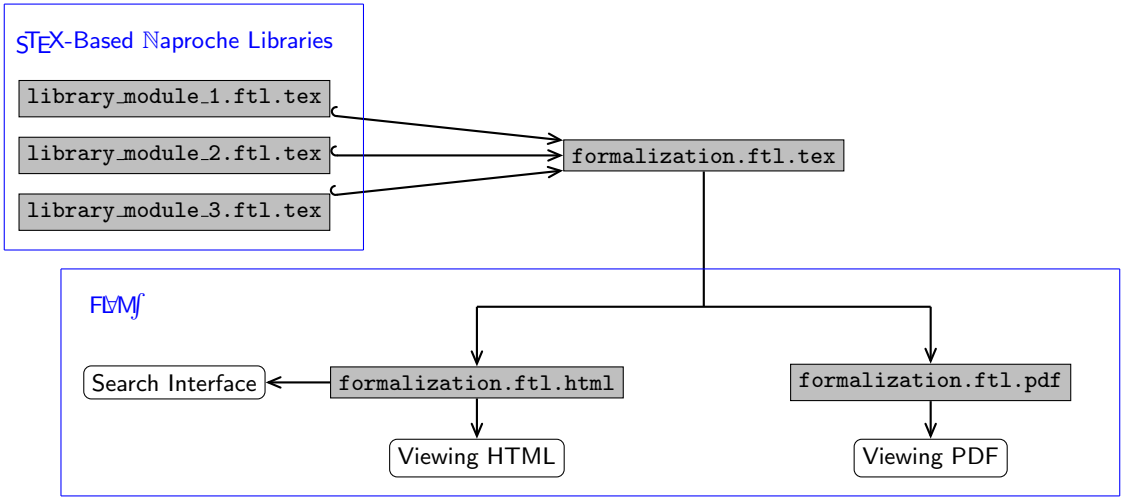
Overview



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Motivation

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Motivation

NatFoM 2023:

Introduced libraries of ℕaproche formalizations

⇒ Focussed on the "logical level"

Remaining Challenges on the "Presentational Level":

For Formalization Authors:

- Importing TEX macro definitions from a library
- Referencing assertions stated in a library
- Searching symbols/notions/assertions across libraries

For Formalization Readers:

- Retrieving definitions/statements of symbols/notions/assertions imported from a library

Solution Approach: Integrating sTEX and FLAMS into the ℕaproche ecosystem

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sTEX

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sTeX: A Module System for LaTeX Documents

https://github.com/slatex/stex

- sTeX provides a module system for LaTeX documents
  ⇒ Structure ℕaproche libraries as sTeX modules
- TeX macros defined sTeX modules can be reused in other LaTeX documents
  ⇒ Define ℕaproche symbols as sTeX macros
- Assertions defined in sTeX modules can be referenced in other LaTeX documents
  ⇒ Provide ℕaproche assertions as components of sTeX modules

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sTEX: ℕaproche Libraries as sTEX Modules

```
\begin{smodule}{congruency.ftl}

  \importmodule[libraries/arithmetics]{modulo.ftl}

  \symdef{Cong}[args=3]{#1 \equiv #2 (\text{mod} #3)}

  \symdecl*{congruency of sum}

  \begin{definition}[forthel, for=Cong]
    Let $n, m, k$ be natural numbers such that
    $k \NotEq \Zero$.
    $\Cong{n}{m}{k}$ iff $n \Mod k \Eq m \Mod k$.
  \end{definition}

  \begin{lemma}[forthel, name=congruency of sum]
    Let $n, k$ be natural numbers such that
    $k \NotEq \Zero$.
    Then $\Cong{n \Plus k}{n}{k}$.
  \end{lemma}

\end{smodule}
```

Definition. Let n, k be natural numbers such that $k \neq 0$. $n \equiv m \,(\mathrm{mod}\, k)$ iff $n \bmod k = m \bmod k$.

Lemma. Let n, m, k be natural numbers such that $k \neq 0$. Then $n + k \equiv n \,(\mathrm{mod}\, k)$.

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sTEX: Reusing sTEX Macros & Referencing Assertions

Reusing `\Cong` **&** **Referencing** `congruency of sum`**:**

```
\usemodule[libraries/arithmetics]{congruency.ftl}

...
Then $\Cong{\One \Plus p}{\One}{p}$ (by \sn{congruency of sum}).
```

⇒ Interactive HTML Rendering:

Then $1 + p \equiv 1 \bmod p$) (by congruency of sum).

Definition Let n, m, k be natural numbers such that $k \neq 0$.
$n \equiv m \bmod k$) iff $n \bmod k = m \bmod k$.

Then $1 + p \equiv 1 \bmod p$) (by congruency of sum).

Assertion Let n, k be natural numbers such that $k \neq 0$. Then $n + k \equiv n \bmod k$).

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FLAMS: The Flexiformal Annotation Management System

https://github.com/FlexiFormal/FLAMS

FLAMS provides a webinterface to ...

- ... render LaTeX documents as PDF and interactive HTML.

  ⇒ Render ℕaproche formalizations in FLAMS

- ... view PDF and interactive HTML documents.

  ⇒ View ℕaproche formalizations in FLAMS

- ... search sTeX symbols.

  ⇒ Search ℕaproche symbols/notions/assertions in FLAMS

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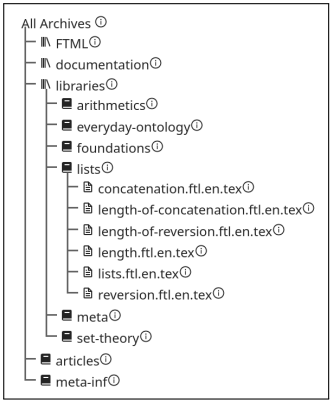
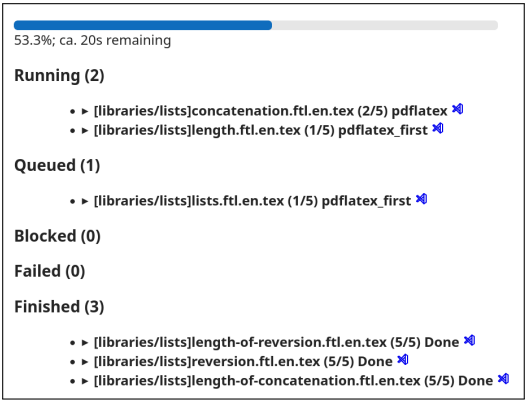
FLAMS: Rendering Formalizations





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FLAMS: Viewing Formalizations

Theorem 18 (Furstenberg). $\mathbb{P}$ is infinite.

Proof. $\cup P$ is a subset of $\mathbb{N}$. Let us show that for any $n \in \mathbb{N}$ we have $n \in \cup P$ iff n has a prime divisor. Let $n \in \mathbb{N}$. If n has a prime divisor then n belongs to $\cup P$.

Proof. Assume n has a p
have $N_{0,p} \in P$. He
If n belongs to $\cup$

Definition 2. $\mathbb{P}$ is the class of all prime numbers.

Definition 15. $P = \{N_{0,p} \mid p \in \mathbb{P}\}$.

Proof. Assume that n belongs to $\cup P$. Take a prime number r such that $n \in N_{0,r}$. Hence $n \equiv 0 \pmod{r}$. Thus $n \bmod r = 0 \bmod r = 0$. Therefore r is a prime divisor of n. □

End. Hence For all $n \in \mathbb{N}$ we have $n \in \cup P^{\complement}$ iff n has no prime divisor. 1 has no prime divisor and any natural number having no prime divisor is equal to 1. Therefore $\cup P^{\complement} = \{1\}$. Indeed $\cup P^{\complement} \subset \{1\}$ and $\{1\} \subset \cup P^{\complement}$. P is infinite.

Proof by contradiction. Assume that P is finite. Then $\cup P$ is closed and $\cup P^{\complement}$ is open. Take a p such that $N_{1,p} \subset \cup P^{\complement}$. $1 + p$ is an element of $N_{1,p}$. Indeed $1 + p \equiv 1 \pmod{p}$ (by congruency of sum). $1 + p$ is not equal to 1. Hence $1 + p \notin \cup P^{\complement}$. Contradiction. □

■

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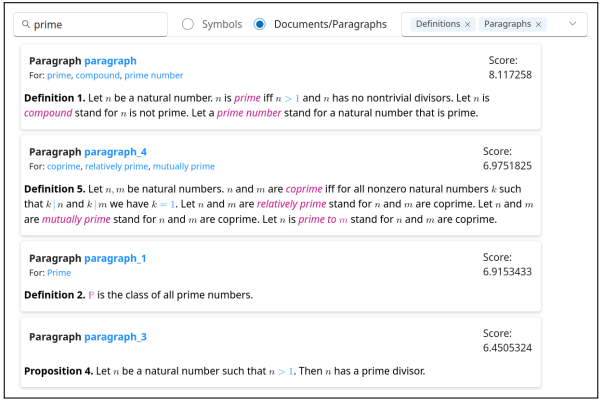
FLAMS: Searching Symbols/Notions/Assertions



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Conclusion

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Conclusion: Managing ℕaproche Formalizations with sTEX and FLAMS

Challenges wrt. Naproche Libraries:

- Importing TEX macros and referencing assertions from a ℕaproche library

  ⇒ Structure ℕaproche libraries as sTEX modules

- Searching symbols/notions/assertions across libraries

  ⇒ Use the search interface of FLAMS

- Retrieving definitions/statements of symbols/statements

  ⇒ Use FLAMS to convert formalizations to HTML that shows definitions/statements on mouse hover

Links:

ℕaproche: https://github.com/naproche/naproche/

sTEX: https://github.com/slatex/stex

FLAMS: https://github.com/FlexiFormal/FLAMS

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