Reaping the Benefits of Modularization in Flexiformal Mathematics by GF-based AST Transformations

Josefin Kelber[2], Michael Kohlhase[1], Jan Frederik Schaefer[1], Marcel Schütz[1]

[1] FAU Erlangen-Nürnberg, [2] Technical University of Munich

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Overview

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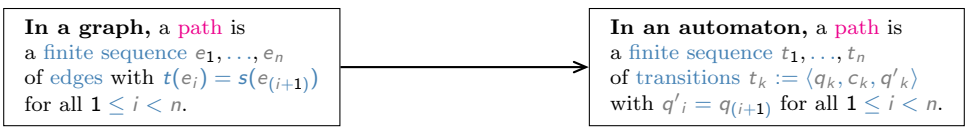

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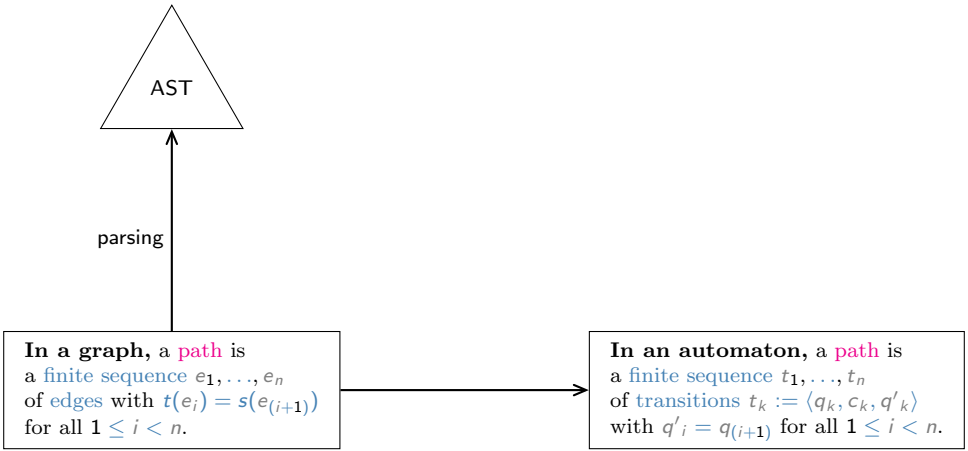

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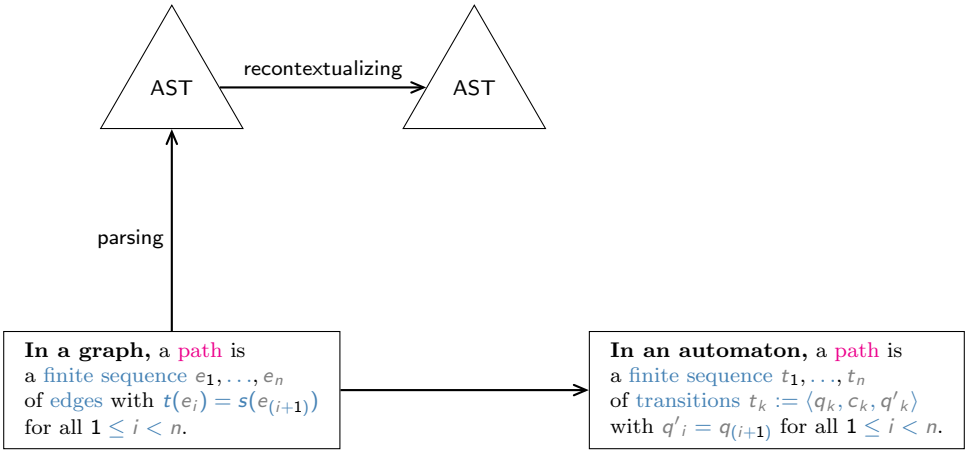

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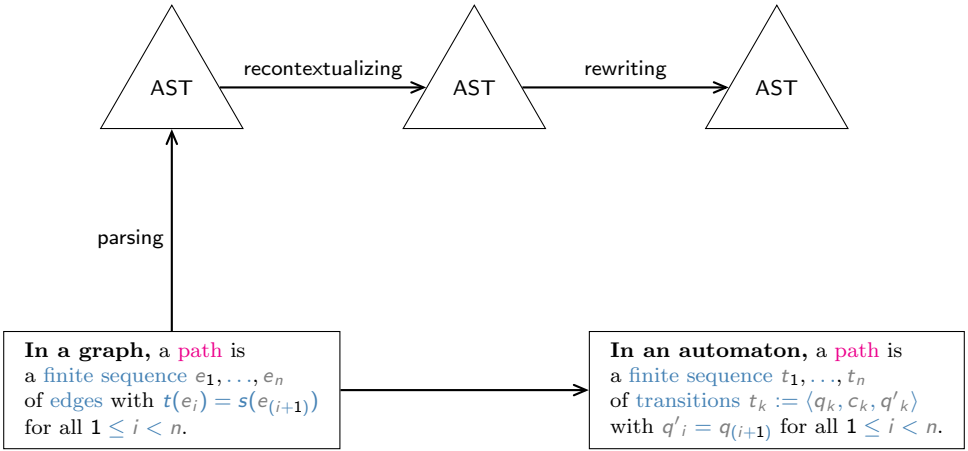

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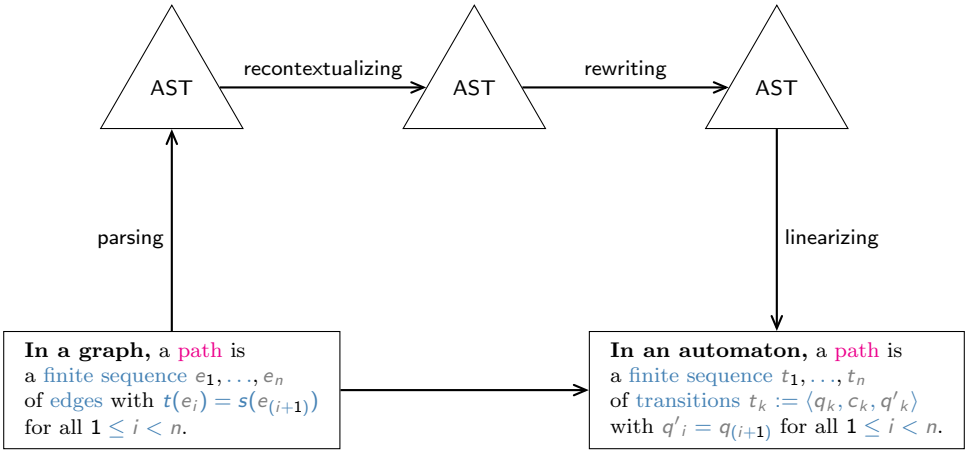

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Motivation

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Motivation: Adaptive Learning Assistants

University Education:

- many students with diverse backgrounds
- few teachers

$\Rightarrow$ difficult to provide individualized learning material

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Adaptive Learning Assistants:

- can track student's learning levels
- can adapt learning material accordingly

$\Rightarrow$ can complement teachers

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Motivation: The ALeA System



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reachable

Definition 0.0. A path is a finite sequence $e_1,..,e_n$ of edges e_i with $t(e_i)=s(e_{(i+1)})$ for all $1 \leq i < n$.

Definition 0.0. A state x is said to be reachable from a state y iff there exists a path from y to x.

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Recontextualization

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Recontextualization: Translation Along Theory Morphisms

Labelled Graph			
V	(nodes)	:	set
E	(edges)	:	set
L	(labels)	:	set
s	(source)	:	$E \to V$
t	(target)	:	$E \to V$
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Automaton

Q	(states)	:	set
Σ	(symbols)	:	set
δ	(trans. rel.)	:	$Q \times \Sigma \to \mathcal{P}(Q)$
q_0	(init. state)	:	Q
F	(fin. states)	:	$\mathcal{P}(Q)$

A transition is a triple $\langle q, c, q' \rangle$ with $q' \in \delta(q,c)$.

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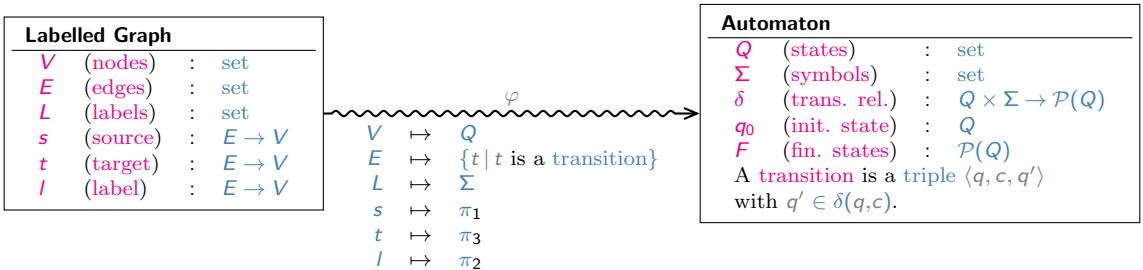

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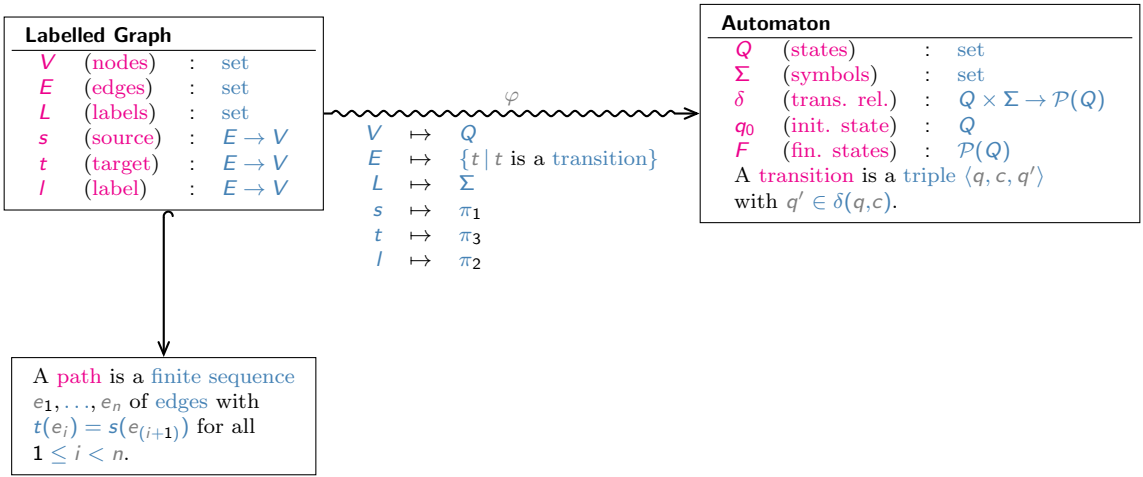

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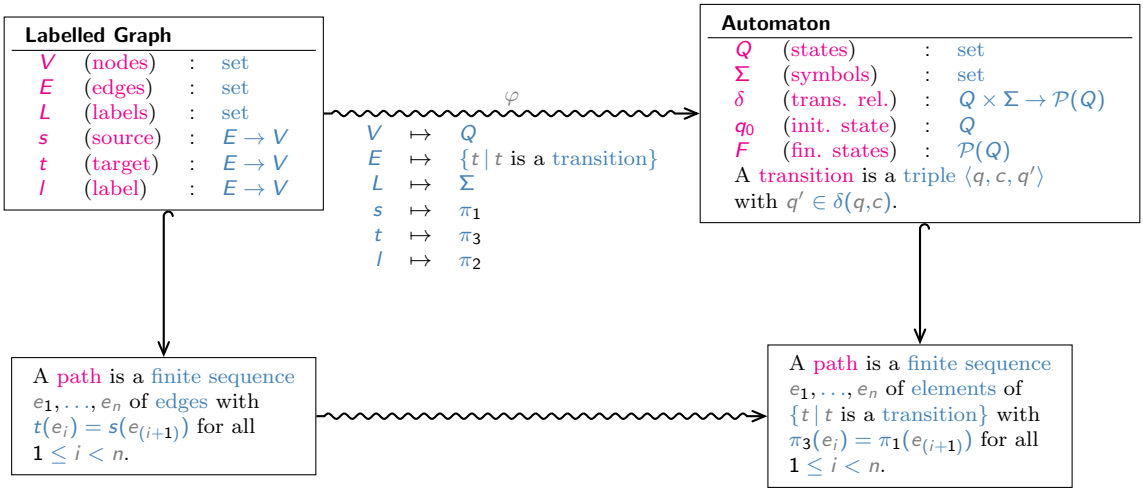

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Recontextualization: Implementation Approaches

String-Based Translation:

A path is a finite sequence $e_1, \ldots, e_n$
of edges with $t(e_i) = s(e_{(i+1)})$ for all $1 \leq i < n$.



A path is a finite sequence $e_1, \ldots, e_n$
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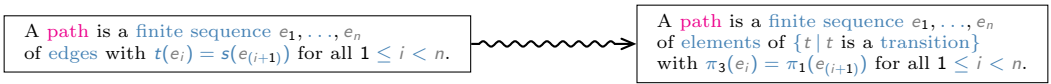
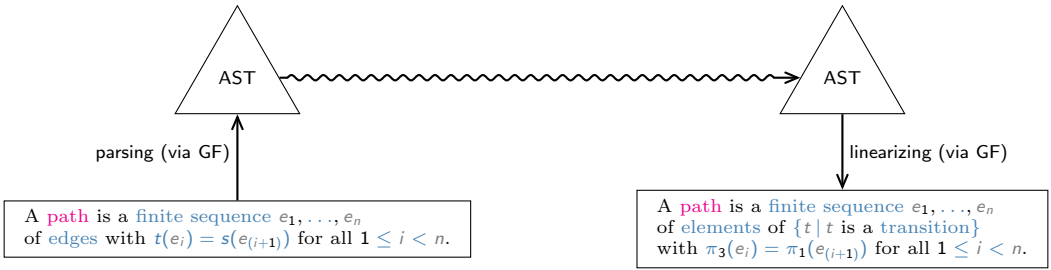


AST-Based Translation:



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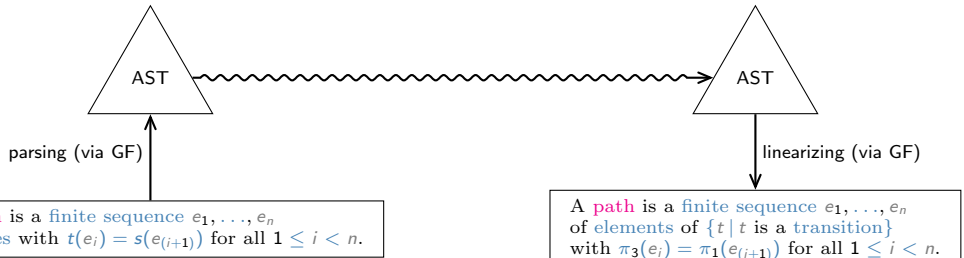

$\Rightarrow$ Grammatical correctness
$\Rightarrow$ Allows rewriting
$\Rightarrow$ Better scalability

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Rewriting

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Rewriting: Applying Natural Language Transformations

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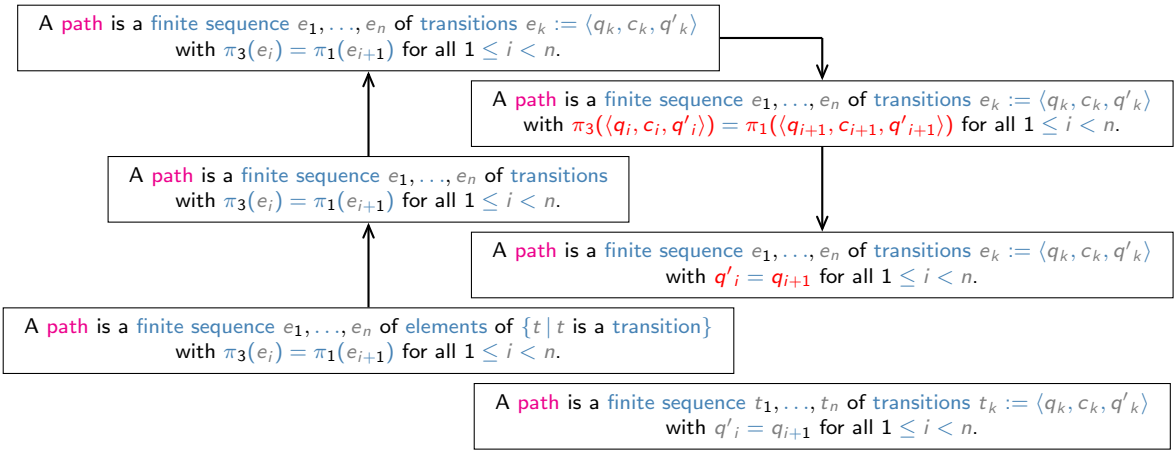

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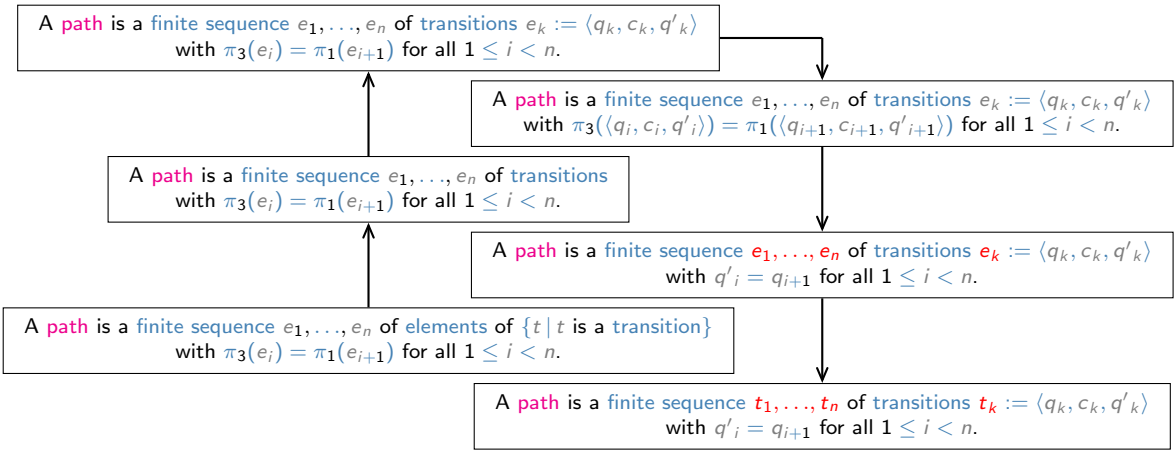

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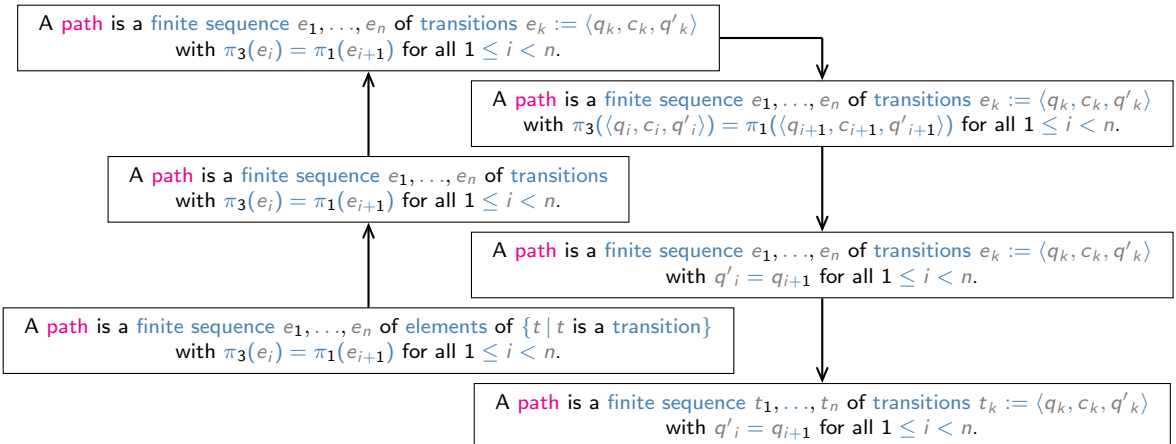

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Rewriting: Natural Language Transformation Rules

Archive of Transformation Rules: https://flexiformal.github.io/rewriting-rules

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"Comprehension Term Reduction":

$$\frac{\begin{array}{rcl}\mathtt{x} & : & [\langle \mathrm{Var}\rangle] \\ \mathtt{y} & : & \langle \mathrm{Var}\rangle \\ \mathtt{z} & : & \langle \mathrm{CN}\rangle \\ \text{"element } \mathtt{x} \text{ of } \{\mathtt{y} \mid \mathtt{y} \text{ is a } \mathtt{z}\}\text{"} & & \end{array}}{\text{"}\mathtt{z}\ \mathtt{x}\text{"}} \qquad \frac{\begin{array}{rcl}\mathtt{x} & = & \text{""} \\ \mathtt{y} & = & \text{"}t\text{"} \\ \mathtt{z} & = & \text{"transition"} \\ \text{"element of } \{t \mid t \text{ is a transition}\}\text{"} & & \end{array}}{\text{"transition"}}$$

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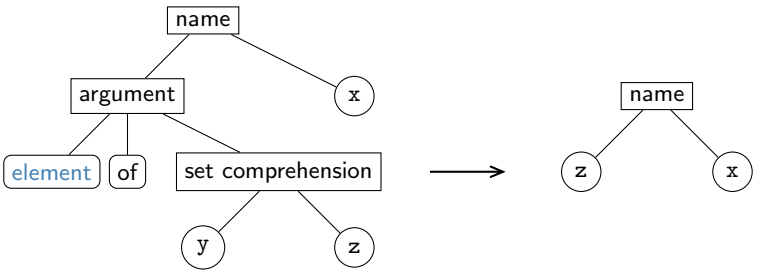

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Conclusion

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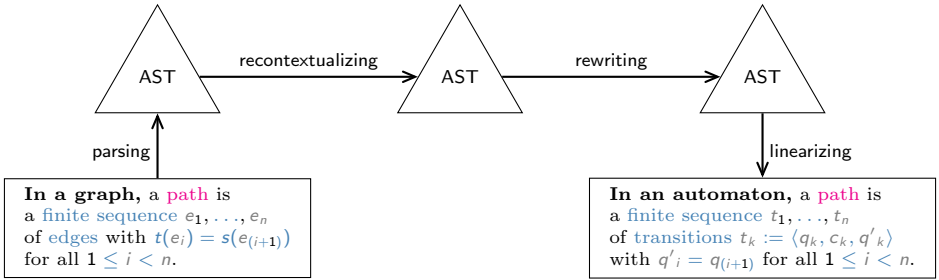
Conclusion: An Improved Recontextualization Mechanism



- Semantic GF-based grammar
- Collection of rewriting rules

⇒ Better scalability

⇒ Grammatical correctness

⇒ Increased naturality

Archive of Rewriting Rules:

https://flexiformal.github.io/rewriting-rules

Proof of Concept Implementation:

https://github.com/FlexiFormal/relocalization

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