

Theory Morphisms in Computer Supported Education

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Theory Morphisms in Computer Supported Education

Current Problems in Education:

- ▶ Many students with different educational backgrounds
- ▶ Few teaching staff

→ Students cannot be supervised individually

Solution Approach:

Use methods from MKM (based on OMDoc/MMT) to ...

- ▶ ... offer individualized online learning services
- ▶ ... auto-generate individualized learning material

The ALEA system

ALEA = **A**daptive **L**earning **A**ssistant

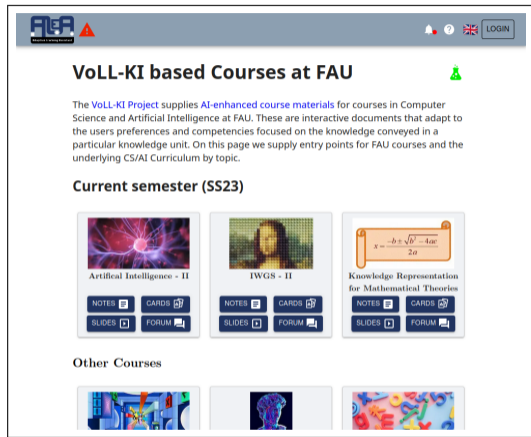
→ Offers (individualized) learning services via MKM technology

Definition 0.1. The probability distribution for a random variable X , written $P(X)$, is the vector of probabilities for the (ordered) domain of X .

▷ **Definition 0.1.** If X is a random variable and x a possible value, we will refer to the fact $X = x$ as an outcome and a set of outcomes as an event. The set of possible outcomes of X is called the domain of X .

Hovering over terms in semantically annotated course material

Tomorrow: Talk about ALEA



The screenshot shows the ALEA web interface. At the top, there is a navigation bar with the ALEA logo, a red triangle icon, and a 'LOGIN' button. Below the navigation bar, the main heading is 'VoLL-KI based Courses at FAU'. A green user icon is visible in the top right corner. The main content area contains a paragraph describing the VoLL-KI Project and its purpose. Below this, there is a section titled 'Current semester (SS23)' which lists three courses: 'Artificial Intelligence - II', 'IWGS - II', and 'Knowledge Representation for Mathematical Theories'. Each course card includes a representative image, the course title, and buttons for 'NOTES', 'CARDS', 'SLIDES', and 'FORUM'. Below the 'Current semester' section, there is a section titled 'Other Courses' which shows three more course cards with representative images.

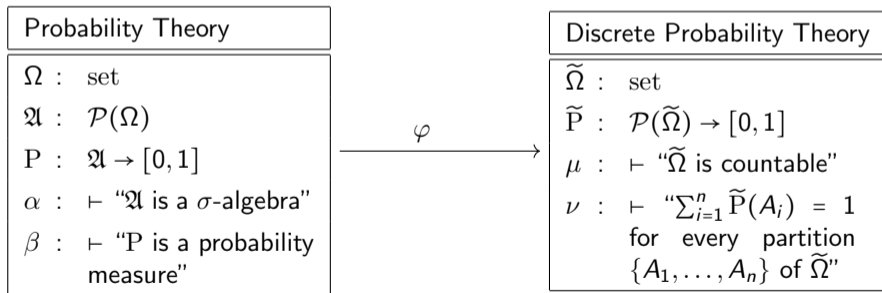
Courses adopted to ALEA

<https://courses.voll-ki.fau.de/>

Theory Morphisms in Computer Supported Education

Definition: A *theory morphism* $\varphi : S \rightarrow T$ between two theories S and T is an interpretation of S -symbols in T such that φ maps all S -theorems to T -theorems.

Example:



Theory Morphisms

Theory morphisms are everywhere:

- ▶ Implementations

(abstract) Euclidean Algorithm \longrightarrow (concrete) Java implementation

- ▶ Examples

Ring $\longrightarrow \langle \mathbb{Z}, +, \cdot \rangle$

- ▶ Functors

π_0 : Group \longrightarrow Pointed Topological Spaces (fundamental group)

- ▶ Equivalent definitions

Discrete probability spaces via probability function \longleftrightarrow
Discrete probability spaces via probability *mass* function

Auto-convert course material to different abstraction/formality levels

- ▶ Existing course material can be reused for lectures at different abstraction/formality levels
- ▶ Students can request presentations of course fragments at, e.g., lower abstraction/formality levels to enhance their understanding of those fragments

Example:

Definition(s). A *random variable* is . . .

1. . . a variable quantity whose value depends on possible outcomes of unknown variables and processes we do not understand.
2. . . a function $X : \mathcal{P} \rightarrow S$ for some discrete probability space \mathcal{P} and some countable set S .
3. . . a function $X : \mathcal{P} \rightarrow \mathcal{M}$ for some probability space \mathcal{P} and some measure space \mathcal{M} .

Making Use of Theory Morphisms

Let S and T be equivalent theories.

Auto-convert course material based on S to (equivalent) material based on T .

Let S and T be equivalent theories.

Lecturers can more easily reuse their course material.

Example:

- ▶ S = Discrete probability theory via probability functions
- ▶ T = Discrete probability theory via probability *mass* functions

Making Use of Theory Morphisms

Automatically provide individualized examples of (abstract) theories

- ▶ Students can request examples that do not exceed their current state of knowledge.
- ▶ Lecturers can request proposals of such examples in the course of creating lecture notes.

Example:

Discrete Probability Theory

Ω : set

P : $\mathcal{P}(\Omega) \rightarrow [0, 1]$



Model of a Fair Coin Flip

Ω := {H, T}

$P(\emptyset)$:= 0

$P(\{H\})$:= 0.5

$P(\{T\})$:= 0.5

$P(\{H, T\})$:= 1

Theory Morphisms in Computer Supported Education

ALeA:

→ Offering individualized learning services

Theory morphisms:

→ Auto-generate individualized learning material

<https://courses.voll-ki.fau.de/>