Theory Morphisms in Computer Supported Education

Marcel Schütz

FAU Erlangen-Nürnberg

6 September 2023

CICM 2023, Cambridge, UK, 4–8 September 2023

Doctoral Symposium

Theory Morphisms in Computer Supported Education

Computer Supported Education

Current Problems in Education:

- Many students with different educational backgrounds
- Few teaching staff

→ Students cannot be supervised individually

Solution Approach:

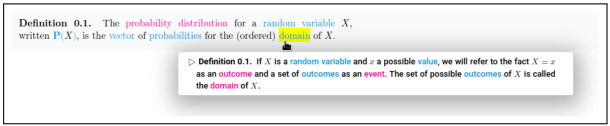
Use methods from MKM (based on OMDoc/MMT) to ...

- ... offer individualized online learning services
- ... auto-generate individualized learning material

The ALeA system

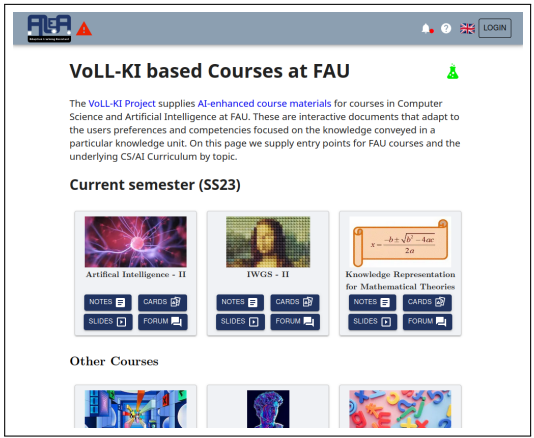
ALeA = **A**daptive **Le**arning **A**ssistant

→ Offers (individualized) learning serices via MKM technology



Hovering over terms in semantically annotated course material

Tomorrow: Talk about ALeA



Courses adopted to ALeA

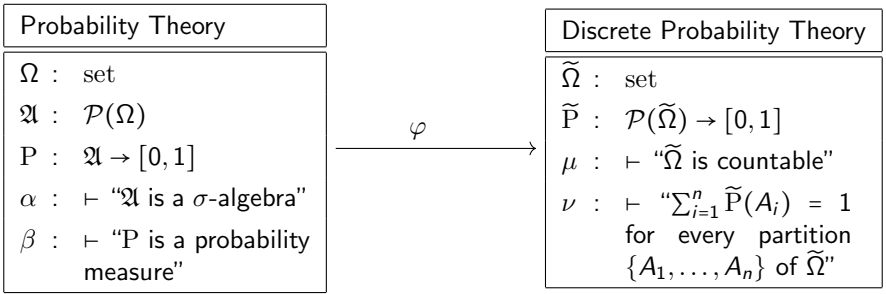
https://courses.voll-ki.fau.de/

Theory Morphisms
in
Computer Supported Education

Theory Morphisms

Definition: A *theory morphism* $\varphi : S \to T$ between two theories S and T is an interpretation of S-symbols in T such that φ maps all S-theorems to T-theorems.

Example:



Theory Morphisms

Theory morphisms are everywhere:

- Implementations

  (abstract) Euclidean Algorithm $\longrightarrow$ (concrete) Java implementation

- Examples

  Ring $\longrightarrow \langle \mathbb{Z}, +, \cdot \rangle$

- Functors

  $\pi_0 :$ Group $\longrightarrow$ Pointed Topological Spaces (fundamental group)

- Equivalent definitions

  Discrete probability spaces via probability function $\longleftrightarrow$
  Discrete probability spaces via probability *mass* function

Making Use of Theory Morphisms

Auto-convert course material to different abstraction/formality levels

- Existing course material can be reused for lectures at different abstraction/formality levels
- Students can request presentations of course fragments at, e.g., lower abstraction/formality levels to enhance their understanding of those fragments

Example:

Definition(s). A *random variable* is ...

1. ... a variable quantity whose value depends on possible outcomes of unknown variables and processes we do not understand.
2. ... a function $X : \mathcal{P} \to S$ for some discrete probability space $\mathcal{P}$ and some countable set S.
3. ... a function $X : \mathcal{P} \to \mathcal{M}$ for some probability space $\mathcal{P}$ and some measure space $\mathcal{M}$.

Making Use of Theory Morphisms

Let S and T be equivalent theories.

Auto-convert course material based on S to (equivalent) material based on T.

Let S and T be equivalent theories.

Lecturers can more easily reuse their course material.

Example:

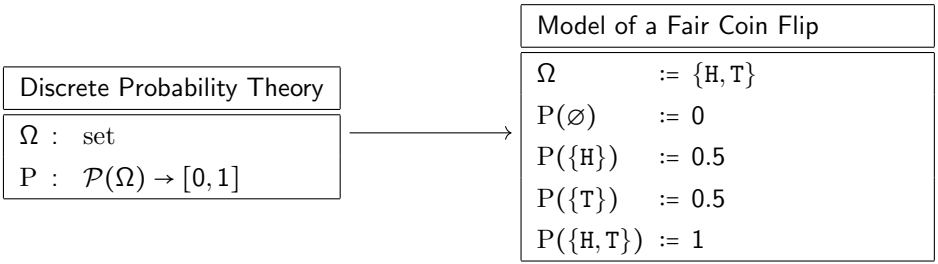
- S = Discrete probability theory via probability functions
- T = Discrete probability theory via probability *mass* functions

Making Use of Theory Morphisms

Automatically provide individualized examples of (abstract) theories

- Students can request examples that do not exceed their current state of knowledge.
- Lecturers can request proposals of such examples in the course of creating lecture notes.

Example:



Conclusion

Theory Morphisms in Computer Supported Education

ALeA:

→ Offering individualized learning services

Theory morphisms:

→ Auto-generate individualized learning material

https://courses.voll-ki.fau.de/