

Reusing Learning Objects via Theory Morphisms

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Learning Objects

What is a Learning Object?

Theorem. Propositional logic $\langle \models_{\text{PL}^0}, \mathcal{L}_{\text{PL}^0}, \mathcal{K}_{\text{PL}^0} \rangle$ naturally forms a logical system $\langle \mathcal{L}, \mathcal{M}, \models, \neq \rangle$.

Definition. Let A be a formula of propositional logic. A is **satisfiable** iff there exists a **model** \mathcal{I} such that $\mathcal{I}(A) = \text{T}$.

Exercise. Is the formula $p \wedge (q \Rightarrow \neg p)$ satisfiable?

Yes

No

Definition. A **logical system** is a triple $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$, where

- ▶ \mathcal{L} is a **formal language**, whose elements are called **formulas**,
- ▶ \mathcal{M} is a **set**, whose elements are called **models**,
- ▶ $\models \subseteq \mathcal{M} \times \mathcal{L}$.

Definition. **Propositional logic** is the triple $\langle \models_{\text{PL}^0}, \mathcal{L}_{\text{PL}^0}, \mathcal{K}_{\text{PL}^0} \rangle$, where

- ▶ $\mathcal{L}_{\text{PL}^0}$ is the **set** of all **propositional formulas**,
- ▶ $\mathcal{K}_{\text{PL}^0}$ is the **set** of all **Boolean interpretation functions**,
- ▶ $\mathcal{I} \models_{\text{PL}^0} A$ iff $\mathcal{I}(A) = \text{T}$.

Theorem. Propositional logic $\langle \models_{\text{PL}^0}, \mathcal{L}_{\text{PL}^0}, \mathcal{K}_{\text{PL}^0} \rangle$ naturally forms a logical system $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$.

Recontextualizing Statements

Logical Systems:

Definition. Let F be a formula of a logical system. A model \mathcal{M} satisfies F iff $\mathcal{M} \models F$.



Propositional Logic:

Definition. Let A be a formula of propositional logic. A model \mathcal{I} satisfies A iff $\mathcal{I} \models_{\text{PL}^0} A$.

Definition. Let A be a formula of propositional logic. A model \mathcal{I} satisfies A iff $\mathcal{I}(A) = \text{T}$.

Recontextualizing Exercises

Logical Systems:

Exercise. Is the formula $F := \text{definiens}(F)$ satisfiable?

- Yes
- No

Feedback: Actually, there is a model \mathcal{M} that satisfies F :
 $\text{definiens}(\mathcal{M})$. Then $\text{conclusion}(\Phi)$:
Indeed, $\text{proof}(\Phi)$.

Propositional Logic:

Exercise. Is the formula $A := p \wedge (q \Rightarrow \neg p)$ satisfiable?

- Yes
- No

Feedback: Actually, there is a model \mathcal{I} that satisfies A :

It maps p to \top and q to F . Then $\mathcal{I} \models_{\text{PL}^0} A$:

Indeed, this can directly be seen by evaluating the truth table for A .

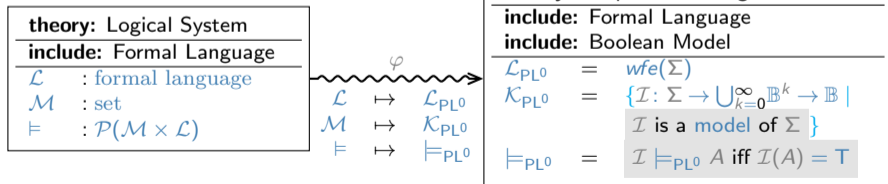
Theory Morphisms

What is a Theory Morphism?

Definition. A **theory morphism** $\varphi : S \rightsquigarrow T$ between two **theories** S and T is a mapping of the **symbols** in S to **expressions** in T such that

$$s : \tau \quad \Longrightarrow \quad \varphi(s) : \varphi(\tau)$$

for all **symbols** s and **types** τ in S .

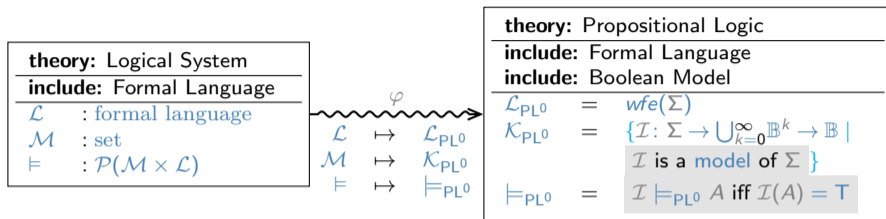


Representing Theory Morphisms in \LaTeX

```
1 \begin{mathstructure}{logsys}[logical system]
2   ...
3   % The satisfaction relation:
4   \symdef{satrel}[
5     args=2,
6     type={\powerset{\cart{\modcls,\flang}}}
7   ]{#1\vDash#2}
8   ...
9 \end{mathstructure}
```

```
1 \begin{mathstructure}{proplog}[propositional logic]
2   ...
3   % The satisfaction relation:
4   \symdef{psat}[
5     args=2,
6     def={\textrm{\$psat{\Ivar!}{\Avar}$ iff $\eq{\Ivar{\Avar},\semtrue}$}}
7   ]{#1\vDash_{\textrm{PL}}^0#2}
8   ...
9
10  % The theory morphism from logical systems to propositional logic:
11  \begin{realization}{logsys}
12    ...
13    \assign{satrel}{\psat!}
14    ...
15  \end{realization}
16 \end{mathstructure}
```

Presenting Theory Morphisms

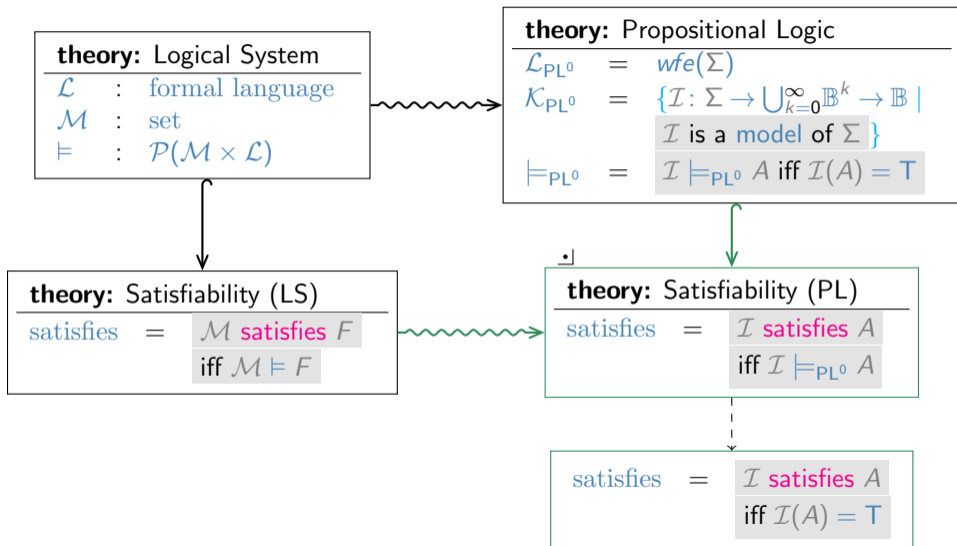


Theorem. \langle target concept \rangle \langle target structure \rangle naturally forms a \langle source concept \rangle \langle source structure \rangle .

Theorem. \langle Propositional logic \rangle $\langle \models_{PL^0}, \mathcal{L}_{PL^0}, \mathcal{K}_{PL^0} \rangle$ naturally forms a \langle logical system \rangle $\langle \mathcal{L}, \mathcal{M}, \models, \not\models \rangle$.

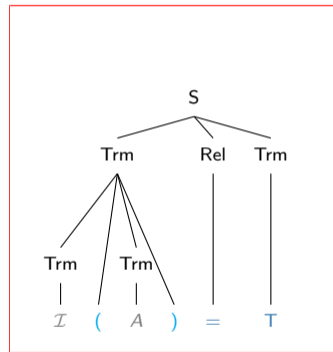
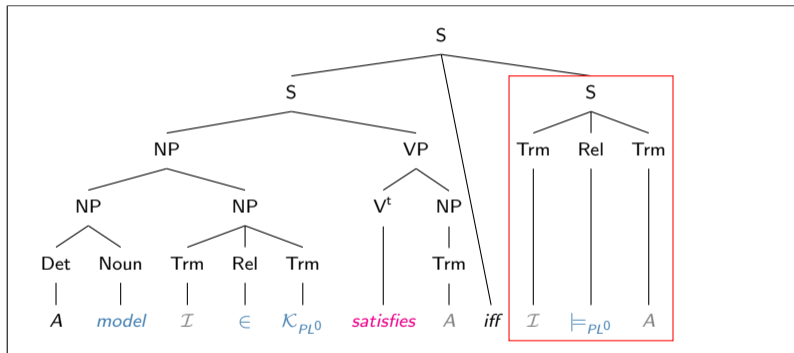
Recontextualizing Learning Objects Along Theory Morphisms

Recontextualizing Statements (Revisited)



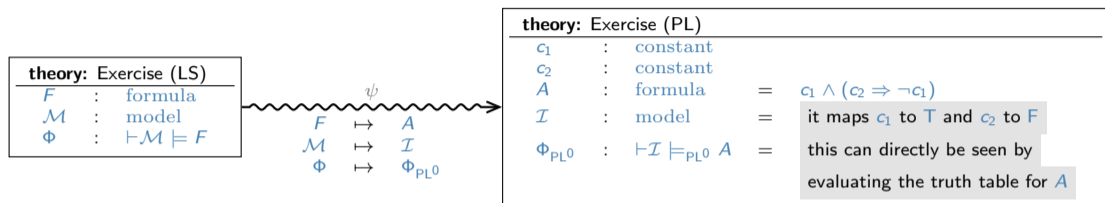
Definition Expansion

“A model $\mathcal{I} \in \mathcal{K}_{PL^0}$ satisfies A iff $\mathcal{I} \models_{PL^0} A$.”



“A model $\mathcal{I} \in \mathcal{K}_{PL^0}$ satisfies A iff $\mathcal{I}(A) = T$.”

Recontextualizing Exercises (Revisited)



Exercise. Is the formula $F := \text{definiens}(F)$ satisfiable?

- Yes
- No

Feedback: Actually, there is a model \mathcal{M} that satisfies F :

$\text{definiens}(\mathcal{M})$. Then $\text{conclusion}(\Phi)$:

Indeed, $\text{proof}(\Phi)$.

Conclusion and Future Work

Reusing Learning Objects via Theory Morphisms

- ▶ We can **automatically** ...
 - ▶ ... present **theory morphisms** in natural language.
 - ▶ ... **recontextualize** statements along **theory morphisms**.
 - ▶ ... **recontextualize** exercises with solutions and feedback along **theory morphisms**.
→ <https://gitos.rrze.fau.de/voll-ki/fau/system/relocalization/>
- ▶ We have a growing corpus of **theory morphisms** and templates that are suited for automatic **recontextualization**.
→ E.g. <https://gl.mathhub.info/courses/FAU/AI/problems/-/tree/main/source/csp/prob>
- ▶ **Future Work:**
 - ▶ Properly handling the intricacies of natural language.
 - ▶ Integrating the **recontextualization** processes in our adaptive learning assistant **ALEA**.
→ <https://courses.voll-ki.fau.de/>